Harmonic Wand: An Instrument for Microtonal Control and Gestural Excitation

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ABSTRACT

The Harmonic Wand is a transducer-based instrument that combines physical excitation, synthesis, and gestural control. Our objective was to design an instrument that affords exploratory modes of interaction with the performer's surroundings, as well as precise control over microtonal pitch content and other concomitant parameters. The instrument is comprised of a hand-held wand, containing two piezo-electric transducers affixed to a pair of metal probes. The performer uses the wand to physically excite surfaces in the environment and capture resultant signals. Input materials are then processed using a novel application of Karplus-Strong synthesis, in which these impulses are imbued with discrete resonances. We achieved gestural control over synthesis parameters using a secondary tactile interface, consisting of four force sensitive resistors (FSR), a fader, and momentary switch. As a unique feature of our instrument, we modeled pitch organization and associated parametric controls according to theoretical principles outlined in Harry Partch's "monophonic fabric" of Just Intonation—specifically his conception of *Odentities*, *Udentities*, and a variable *Numerary Nexus*. This system classifies pitch content based upon intervallic structures found in both the overtone *and* undertone series. Our paper details the procedural challenges in designing the Harmonic Wand.

Author Keywords

Microtonal, Gestural Control, Physical Excitation, Karplus-Strong

CCS Concepts

• **Applied computing → Sound and music computing**; *Performing arts*; • **Hardware →** *Sensors and Actuators*;

1. INTRODUCTION

Our objective is to design an instrument which, in performance, combines the exploratory modes of interaction afforded through direct engagement with the physical environment and the precise control of pitch content necessary to accurately recreate various microtonal or Just tuning systems, as well as other spectrally-derived sonorities. These two objectives suggest a linkage between physical and virtual forms of interaction. From a physical standpoint, we have sought the use of a pair of input transducers as a means of gathering acoustical materials from surfaces within reach of the performer. Both transducers are attached to opposing ends of a single, bifurcated handle—or "impulse wand"—which the performer grips, using his or her right-hand to probe surrounding surfaces. These input materials are processed using a novel extension of Karplus-Strong synthesis, in which acoustic textures collected from the environment are treated as impulses and imbued with discrete resonances. The frequency of each resonance is assigned from a collection of pitches contained within a user-defined tuning system. To categorize and structure pitch materials, we chose to apply Harry Partch's conception of Just Intonation—what he refers to as a "Monophonic Fabric" of *Otonalities*, *Utonalities*, *Odentities*, *Udentities*, and *Numerary Nexus*. Consequently, decisions governing interface design, functionality, and performance practice are informed by principles outlined in Partch's major treatise, *Genesis of a Music* [12]. Control over the tuning structure, spectra, and envelope are achieved using a tactile interface, operated with the left-hand.

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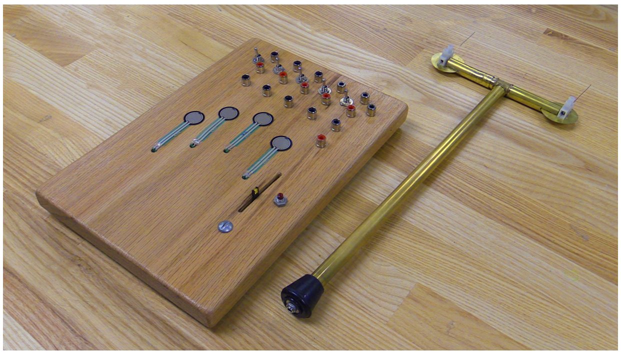

Figure 1. Impulse Wand and Tactile Interface.

2. RELATED WORK

Similar applications of object-specific or surface-based transduction have been proposed by Alex Baker, Hugh Davies, Richard Lerman, Eric Leonardson, Otomo Yoshihide, and Ivan Palacky [3]. Procedurally, the application of dual piezo transducers within the impulse wand certainly shares a similar emphasis on exploratory practices and material properties of found objects with Eric Leonardson's "springboard" performances [7]. Preceding this work, a multi-channel approach to piezo input transduction and analogous privileging of physical texture and materiality in performance can be seen as early as 1968 with Hugh Davies' instrument, the "Shozyg" [9]. Sharing a similar performance modality, Merrill, Raffle, and Aimi's "Sound of Touch" also employs a wand-like device to stimulate physicals surfaces and digitally process the resultant signals—in this case, via convolution [10]. Though many of these earlier instruments pair time or frequency-based signal processing with piezo transduction, we have yet to encounter a precedent for coupling Karplus-Strong synthesis with found or incidental source materials, nor an explicit emphasis on reconciling exploratory transduction and tuning practices.

3. DESCRIPTION

The Harmonic Wand is modular in design, consisting of two primary components: an impulse wand and tactile interface. While the impulse wand acts as an input-transducer, capturing minute signals from the environment via physical excitation, the secondary tactile interface behaves as a modifier, altering the values of various parameters in response to gestural movements from the left-hand[1]. Signals captured with the wand are fed through a series of Karplus-Strong synthesis algorithms (programmed in Pure Data), where they are treated as a continuous stream of impulses.

3.1 Impulse Wand

In designing an appropriate device for encouraging exploratory behaviors, we considered a range of human motion and gesture, as well as the ability to transduce signals directly from a variety of surfaces in the environment. To address these dual functionalities, we chose to implement a single, right-hand operated input device or "impulse wand." This configuration frees the performer's left-hand to manipulate other parameters. Meanwhile, the right-hand engages in exploratory behaviors dealing in touch, texture, and space. Though this division of motions could certainly be reversed, or perhaps arranged in a decidedly ambidextrous fashion, the current layout functions suitably.

In construction of the device, we utilized readily available parts and materials. The handle of the wand is constructed from a 30-centimeter segment of brass tubing (12-millimeter diameter). An interlocking T-joint connects two shorter, 5-centimeter sections of brass pipe, while two 15-centimeter slots at either side accommodate insertion of two 35-millimeter piezo discs. One section of terminal strip is affixed near the center of each piezo disc using a fast-drying epoxy. Within each terminal strip, an 8-centimeter length of 24-gauge (AWG) wire is secured using a pair of tension-screws. As described by Nicolas Collins, the use of terminal strips has proven an effective means of affixing materials to piezo-electric transducers [3]. During performance, the exposed section of wire acts as a "probe," making transverse contact with surfaces in the environment and allowing the capture of nearly inaudible sounds and vibrations. Signal output from each transducer is routed through the body of the wand to a 3.5-millimeter, stereo audio-jack. As to maintain relative isolation between the two audio sources, the signals are divided and fed into the first two analog inputs of an audio interface. In prototyping this component, we utilized a Motu Ultralite (Mk-1), though any similar device with two analog to digital converters and pre-amplifiers capable of accepting an instrument level signal would suffice.

3.2 Application of Karplus-Strong Synthesis

Direct engagement with frequency content and spectra are key components of the instrument's design. In this respect, the interrelation between the fundamental frequency and constituent partials define both spectra *and* tuning, with harmonic relations expressed through the archetypal form of integral ratios. In exploring the upper partials—specifically those exceeding the seventh, eleventh, or thirteenth harmonic—comes the inclusion of microtonal intervals. To facilitate access to these nuances of pitch embodied in the upper reaches of the harmonic series, we have extended the basic synthesis model to include two recursive delay-lines, or *Karplus-Strong* (KS) *Operators*, running in series (Figure 1). The first KS operator generates a prominent fundamental frequency and a series of ascending overtones. Acting as a resonator, the second KS operator establishes an additional target frequency tuned to a single harmonic partial derived from the initial overtone series, herein referred to as the *Resonant Harmonic Frequency*.

Working in conjunction with audio captured via the "impulse wand," control over the synthesized spectra encompasses four pairs of discrete parameters: *Fundamental Frequency*, *Resonant Harmonic Frequency*, *Harmonic Decay Time* (expressed in milliseconds), and *Filter Cutoff*. As the two impulse signals together constitute a stereoscopic image, each chain of parameters is indicative of a single channel. Each channel therein consists of two KS structures operating in series with the delay-time for the first operator determined by the fundamental frequency value and the delay-time for the second operator expressed as a function of the Resonant Harmonic Frequency value. This value may be transposed by multiple octaves to yield a delay-time no greater than twice the duration of the corresponding delay-time defined by the fundamental frequency. The purpose of this transposition is to allow the propagation of input artifacts whose spectral content is lower in frequency than that of the Resonant Harmonic Frequency. Arranged in series, the lengthened delay-line acts as a waveguide whose duration is an integral multiple of the fundamental frequency's wave period [6]. Consequently, the resultant timbre retains a prominent spectral peak consistent with the Resonant Harmonic Frequency.

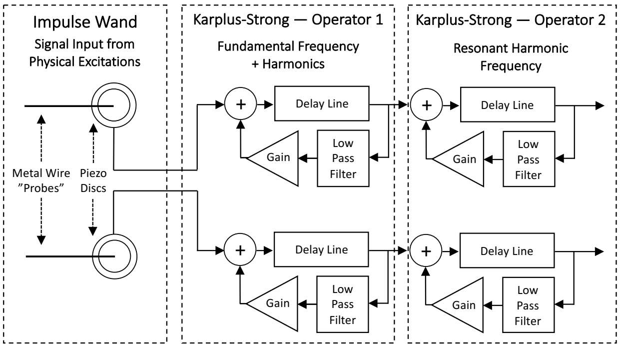

Figure 2. Two Karplus-Strong Operators in Series

3.3 Tactile Interface

The performable parameters described above are accessed using a tactile interface. Mirroring the contours and dimensions of an out-stretched hand, two-centimeter force-sensitive resistors (FSR) are positioned under the fingertips of each of the first four digits of the left-hand. The body of the interface consists of an 18 × 25 × 2-centimeter section of solid oak. Connections between the tactile interface and microcontroller are maintained using RCA interconnects. This modular configuration offers the ability to re-route sensor assignments.

Developing an effective means of interpreting sensor output evolved through two stages of prototyping. During the first stage, we utilized analog sensor inputs from the Bela IO/BeagleBone Black platform. Benefits of this embedded computing device include low-latency inputs—as low as 100 µs, according to the manufacturer—and the ability to compile code

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[1] Performance Demonstration: https://vimeo.com/251231654

from Pure Data using an online IDE [2]. While the portability and low-latency of an embedded computing platform offers a number of advantages, ultimately development shifted to using a laptop for all digital signal processing and an *Arduino Uno* (ATmega328P) microcontroller to process sensor inputs [1]. While the Bela IO does offer significantly lower latency, we noticed a significant improvement in audio quality when using dedicated preamplifiers and analog to digital converters.

As each FSR responds to a range of pressure levels, output values are scaled to a specific range for each respective parameter. Output range for each analog input from the Arduino may vary between 0-1024 discrete values. Using exponential scaling, we mapped this range to Harmonic Decay Time values between 100 and 15,000 milliseconds and Low-Pass Filter Cutoff values between 500 and 22,000 Hertz—with corresponding Filter Coefficient (a_1) values ranging between 0.9 and 0.05, respectively. In performance, increased pressure from the second or fourth digits correlate to a lengthening of Harmonic Decay Time. Concurrently, increased pressure from the first or third digits results in lower coefficient values and decreased attenuation of high frequency content.

While the interface is designed in such a way that allocation of specific parameters is flexible, channel-specific assignment of iterative parameters (e.g. Harmonic Decay Time, Filter Cutoff) are paired to consecutive digits. For example, Harmonic Decay Time and Filter Cutoff values for channel one are assigned to sensors contacting the fifth and fourth digits, while the third and second digits make contact with sensors controlling the same parameters for channel two. As such, each channel maintains independence in regards to envelope duration and filtration, as well as frequency. However, within each channel, changes in Harmonic Decay Time and Filter Cutoff values operate globally, with overall decay times and filter shape varying at the same rate for both delay-lines in series. While linking these parameters in series does restrict the ability to shape certain aspects of synthesis, it is our view that these limitations afford a more intuitive control structure.

4. TUNING STRUCTURE AND CONTROL

By design, all pitched materials are derived directly from a defined overtone series. As such, the proportional relationships between harmonic partials constitute whole-number or integral ratios implicit to intervals described in Just Intonation. Beyond an inherent correlation between pitch and spectra, key aspects of interface design originate in a desire to model Just tuning systems in performance. Our goal is not to simply create an instrument that can perform a variety of microtonal scales, but apply systemic approaches to intervallic structure based upon the explicit language and procedures of Harry Partch's "Monophonic Fabric [12]."

According to this organizational model, just intervals are classified according to numerical properties defined by either a common numerator or denominator within a given set of pitch ratios. Those intervals sharing a common value in their denominator are referred to as *Otonalities*, while intervals presenting a common numerator are described as *Utonalities* [4]. In either case, this common factor defines a *Numerary Nexus* linking intervals sharing a specific tonality. Whereas an Otonality is analogous in intervallic structure to an ascending sequence of harmonic partials from the overtone series, an ascending Utonality produces a mirror image of the series, or *subharmonics*. Within Partch's system, specific identities distinguish all intervals. These identities refer to the numerical value positioned above or below the Numerary Nexus of a given ratio. With Otonalities, we refer to this value as the ratio's *Odentity*, while the identity of an Utonality is defined as it's *Udentity* [12]. Treated as a set of variables, the Numerary Nexus and Odentity or Udentity may define the qualities of a given ratio within any set of just intervals. Consequently, these important variables are assigned parametric controls within the instrument's tactile user interface (see Figure 3).

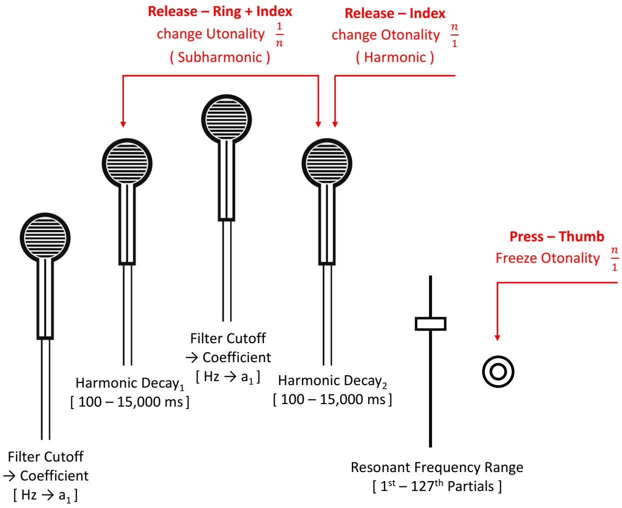

Figure 3. Harmonic Control Structure of Tactile Interface

4.1 Pitch Organization

4.1.1 Udentity

Control of the fundamental frequency for each channel, as defined by the delay-time for KS operator, is a function of its Utonality. Lifting both the second and fourth digits from sensor contacts on the tactile interface generates a change in Udentity corresponding with denominator values of /1, /3, /5, /7, /9, /11, /13, or /15. In performance, the resulting fundamental frequencies consist of a randomized, non-repeating cycle of eight Utonal ratios: 1/1, 4/3, 8/5, 8/7, 8/9, 16/11, 16/13, 16/15 (whereas 1/1 = 27.5 Hertz). By default, the Numerary Nexus for Utonal ratios maintains a constant power of '2' unless otherwise specified. These user-generated fundamental frequencies provide the basis for an overtone series from which a set of Resonant Harmonic Frequency values may be extracted.

4.1.2 Odentity

The structure of ascending intervals in an Otonality mirror the overtone series, with the Numerary Nexus appearing in the denominator and a variable sequence of Odentities occupying the numerator. Treated as integral multiples of the current fundamental frequency (as defined by a specific Udentity), the Odentity establishes a Resonant Harmonic Frequency. Though the system affords access to just intervals embodied within the first 127 partials of the harmonic series, we chose to refine selection of generated Odentities to numbers whose largest prime factor is no greater than thirteen. In this regard our generative approach to just intonation surpasses the scope of Partch's 11-limit threshold. Certainly, 13-Limit tuning systems are not unprecedented. Notably, the work of early twentieth century theorist Kathleen Schlesinger includes 13-limit ratios, as evidenced in the application of this class of intervals within her reconstructions of ancient Greek modes [13]. In addition, Resonant Harmonic Frequency values are further limited to harmonic multiples that are equally divisible by the current Odentity value. This arrangement ensures that generated harmonic frequencies coincide with partials occurring in a single overtone series whose fundamental frequency ratio is always equal to 1/1. Accordingly, all Resonant Harmonic Frequency values are part of a single, unified spectra.

As with the assignment of Udentity parameters, interrupting contact with FSR sensors also triggers a randomized, non-repeating pattern of Odentities—with each instantiation of a new Odentity resulting in distinct Resonant Harmonic Frequency value. However, a change in Odentity value may be initiated by lifting only the fourth digit. This configuration allows the performer to maintain a static fundamental frequency by keeping the second digit in contact with the FSR sensor, while varying the Resonant Harmonic Frequency value by lifting the fourth digit. In addition to FSR sensors, the tactile interface is fitted with a single, 6-centimeter linear fader. Positioned to track a comfortable range of vertical motion from the thumb, signal output from this fader is scaled as to permit the performer to specify a general range of Resonant Harmonic Frequency values to be chosen upon instantiation of a new Odentity. Alternately, when the fader is lowered to a minimum position, frequency values may be randomly chosen from the first 127 harmonic partials of the current Fundamental Frequency value.

4.1.3 Numerary Nexus

Activating a momentary switch located to the right of the linear fader "freezes" the current Odentity, thus maintaining a static numerary nexus. In this mode, the intervallic relationship between the fundamental frequency and corresponding partial remaining constant. This setting affords parallel motion between all subsequently generated pitches. As the switch is momentary, activation of this function is "latched" with a second activation of the switch resulting in a return to variable intervallic motion between the Fundamental and Resonant Harmonic Frequency values.

5. IMPLEMENTING PRECISE CONTROL OF KARPLUS-STRONG RESONATORS

The musical and artistic goals for this instrument require that the performer have precise control of resonant frequencies in each of the KS operators, as well as effective control of the decay time for each operator at both low and high frequencies. These values correlate to the perceptually-based controls which guide the overall Harmonic Decay Time for each channel.

The fundamental resonant frequency, F_R, (or *target pitch*) of a KS operator is a function of the time it takes for audio to circulate through the recursive delay structure. This value can be controlled by setting the length of the delay line according to equation 1. We note that if the delay-line is restricted to an integer number of samples, then control of the pitch is limited. In order to allow more precise control of F_R, we use the delread4~ object in Pure Data. This object implements fractional delay using a four-point FIR interpolation [15].

$$Delay\ Time = F_S/F_R \tag{1}$$

$$where\ F_S = Sampling\ Rate$$

The rate at which energy decays on each KS operator, or Harmonic Decay Time, can be controlled using the feedback gain (ref figure). The Harmonic Decay Time (HDT) value describes the time (in milliseconds) in which the energy from an initial impulse is attenuated by 60 dB (or ≈ 0.001 of the original amplitude). We can also think of this value in terms of the degree of resonance exhibited by the KS operator, wherein a higher resonance corresponds to a longer harmonic decay time. It can be shown that for a KS operator with a resonant frequency (F_R), the feedback gain needed to achieve a specific (HDT) is:

$$Feedback\ Gain = 0.001^{(1000/(HDT \times F_R))} \tag{2}$$

$$\begin{gathered} where \\ F_R = Resonant\ Frequency \\ HDT = Harmonic\ Decay\ Time \end{gathered}$$

Lastly, we intend to control how quickly the energy at high frequencies decays—thus also controlling the overall brightness of the resulting timbres. This function is accomplished using a low-pass filter inside the feedback loop of each KS operator, thereby reducing the energy at high frequencies each time the audio circulates through the loop [8].

Undoubtedly, there are a number of ways to implement such a filter. In his work extending the KS algorithm to electric guitar, Sullivan proposes a symmetric 3-tap FIR filter [15]. This filter has a linear phase response and a group delay of one-sample across all frequencies. The group delay of a filter is equivalent to the time delay added by the filter to each sinusoidal component of the filter's input signal. This constant group delay is a useful property, as we can reduce the delay length by one sample and maintain precise control of the resonant frequency. Unfortunately, we found that the roll-off of this filter is not steep enough to allow the degree of expressive control over high-frequencies that we desire. The range and variation in spectra was rather static, privileging bright timbres and precluding a great deal of expressivity in performance.

In order to achieve more control of high frequencies, we elected to use a DC-normalized, one-pole IIR filter with a roll-off of 6 dB per octave. This filter is controlled by its -3dB cutoff frequency. To achieve a desired cutoff frequency (*f*), the feedback coefficient (a_1) can be calculated from the amplitude response of the filter (see Equation 3) [14].

$$G(f) = \frac{1 - |a_1|}{\sqrt{1 + a_1{}^2 + 2 \times a_1 \times \cos(2\pi f/F_s)}} \tag{3}$$

Unlike the symmetric FIR filter proposed by Sullivan, the phase response of the one-pole filter is not linear, and thus the phase delay varies across frequency (see figure 4). In order to maintain precise control of the resonant frequency we need to compensate for this delay. We can remedy this discrepancy by calculating the phase delay at the desired resonant frequency and then reduce the delay length by this amount (as shown in Equation 4) [14, 15]. Unfortunately, the non-constant phase delay means that the other resonances in the KS operator may occur at ratios that are slightly inharmonic to the fundamental frequency. This is a trade-off we are willing to accept.

$$Phase\ Delay\ at\ Fr = \frac{1}{\omega} \tan^{-1} \left[\frac{a_1 \sin(\omega)}{1 - a_1 \cos(\omega)} \right] \tag{4}$$

$$where\ \omega = 2\pi \left[\frac{F_R}{F_s}\right]$$

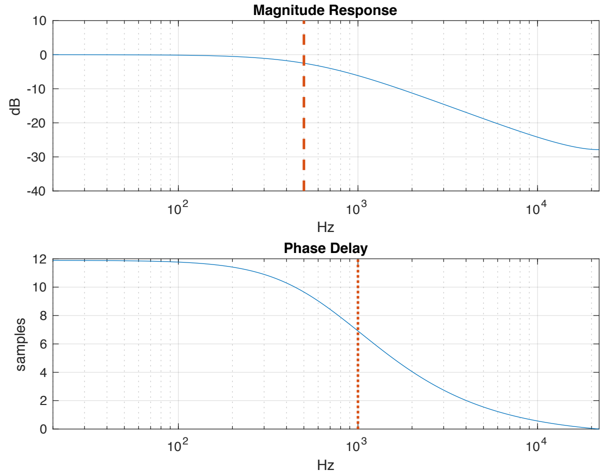

Figure 4: Shows the amplitude response and group delay for the 1-pole low-pass filter when the cutoff frequency is 500 Hz. If, for example, the resonant frequency of the KS-operator is 1kHz, you can see that the filter adds 7 samples of delay to the loop.

6. DISCUSSION

The Harmonic Wand is a new instrument which utilizes physical excitation, Karplus-Strong synthesis, and gestural control to facilitate exploratory modes of interaction, as well as precise control over microtonal tuning structures and other affiliated parameters. In constructing the instrument, we sourced readily available materials and employed sensor, transducer, and microcontroller technologies commonly used in the design of other gestural controllers (e.g. force sensitive resistors, piezo discs). By employing a novel extension of Karplus-Strong synthesis, our project addresses the challenges of mitigating the effects of phase delay and resultant inharmonic artifacts associated with IIR filters.

However, our most unique contributions to the field are combinatory, resulting in a design which integrates physical excitation, gesture, and synthesis with the idiomatic language and procedures of Just Intonation. In turn, we modeled the layout and function of parametric controls within the instrument's tactile interface according to structural principles conceived by microtonal theorist and composer Harry Partch [12]. Globally, these gestures afford the performer access to intervallic subtleties embodied within the natural overtone *and* undertone series, as well as harmonic intersections between these two tonalities. In practice, the first author has performed with instrument on four occasions, including a group improvisation and an original composition for Harmonic Wand and cello (*Rainshadow*). This experience has revealed important performative insights. For example, we observed that independent control over Harmonic Decay Time and Filter Cutoff values affords continuous interpolation between very disparate sonorities, ranging from percussive bursts to sinusoidal pitch clusters and harmonically-rich drones.

It is worth noting that, while our work suggests one viable approach to phase delay compensation, additional research in other filter designs may yield both accurate and aesthetically compelling results. Likewise, future iterations of the instrument may also benefit from independent parametric control over each KS operator. Certainly, the process of addressing alternate methods of pitch organization is fundamental in this instrument's design. Naturally, other historical and contemporary approaches to Just tuning, such as Erv Wilson's "combination product sets" [5], warrant further attention and may yet yield new paradigms in musical interface design. We look forward to investigating these potentials as we continue to develop this instrument.

7. REFERENCES

[1] Arduino Uno ATmega328P Technical Specifications, Arduino https://store.arduino.cc/usa/arduino-uno-rev3 (accessed July 2, 2017).

[2] Bela IO Technical Specifications, Augmented Instruments Laboratory, Queen Mary University of London, http://www.bela.io (accessed Feb. 4, 2017).

[3] N. Collins, *Handmade Electronic Music: The Art of Hardware Hacking*. Second Edition. New York, NY: Routledge, 2009.

[4] B. Gilmour, "Changing the Metaphor: Ratio Models of Musical Pitch in the Work of Harry Partch, Ben Johnston, and James Tenney," *Perspectives of New Music*, Vol. 33, No. 1/2 (Winter-Summer, 1995): 458-503. http://www.jstor.org/stable/833715 (accessed Jan. 3, 2018).

[5] F. Jedrzejewski, "Euler et les Réseaux Harmoniques," *Intersections: Canadian Journal of Music*, Vol. 29, Issue 2, (2009): 66-83. https://search.lib.virginia.edu/articles/article?id=a9h%3A52248437 (accessed Jan. 10, 2018).

[6] K. Karplus and A. Strong, "Digital Synthesis of Plucked-String and Drum Timbres," *Computer Music Journal* 7, no. 2 (1983): 43-55. http://www.jstor.org/stable/3680062 (accessed Apr. 9, 2017).

[7] E. Leonardson, "The Springboard: The Joy of Piezo Disk Pickups for Amplified Coil Springs," *Leonardo Music Journal*, Vol. 17 (2007): 17-20. http://www.jstor.org/stable/4540612 (accessed Dec. 27, 2017).

[8] N. Lindroos, H. Penttinen, and V. Välimäki, "Parametric Electric Guitar Synthesis," *Computer Music Journal* 35, no. 3 (2011): 18-27. http://www.jstor.org/stable/41241762 (accessed Apr. 9, 2017).

[9] J. Mooney, "Hugh Davies' Self-Built Instruments and their relation to Present-Day Electronic and Digital Instrument-Building Practices: Towards Common Themes," Paper delivered at the International Festival for Innovations in Music Production and Composition (IFIMPAC) Conference, Leeds, UK, March 13th, 2015. http://www.jamesmooney.co.uk/publications (accessed Dec. 26, 2017).

[10] D. Merrill, H. Raffle, and R. Aimi, "The Sound of Touch: Physical Manipulation of Digital Sound," *MIT Media Publications,* (April 2008), Cambridge, MA, https://www.media.mit.edu/publications/the-sound-of-touch-physical-manipulation-of-digital-sound/ (accessed March 30, 2018).

[11] G. Moro, S. A. Bin, R. H. Jack, C. Heinrichs, and A. McPherson, "Making high-performance embedded instruments with Bela and Pure Data," *Proceedings of the International Conference on Live Interfaces (ICLI)*, Brighton, UK (2016).

[12] H. Partch, *Genesis of a Music: An Account of a Creative Work, Its Roots, and Its Fulfillments*, New York, NY: Da Capo Press, 1974.

[13] K. Schlesinger, "Further Notes on Aristoxenus and Musical Intervals," *The Classical Quarterly*, Vol. 27, No. 2 (April 1933): 88-96. http://www.jstor.org/stable/6363812 (accessed Dec. 28, 2017).

[14] J.O. Smith, *Introduction to Digital Filters*, Second Edition: Stanford, CA: BookSurge Press/CCRMA, 2008.

[15] C.R. Sullivan, "Extending the Karplus-Strong Algorithm to Synthesize Electric Guitar Timbres with Distortion and Feedback," *Computer Music Journal* 14, no. 3 (1990): 26-37. http://www.jstor.org/stable/3679957 (accessed Apr. 9, 2017).